The Solar Eclipse Inverse Problem

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Case Western Reserve University

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NSF NASA Partner A R D C AMATEUR RADIO DIGITAL COMMUNICATIONS HamSCI CASE WESTERN RESERVE UNIVERSITY

Outline

1. Measuring the Ionosphere
2. Solar Eclipses
   - 2023 Eclipse Data
   - 2024 Eclipse Data
3. Inverse Problems
4. Bayesian Methods
   - Bayes Formula
   - Bayesian Filtering

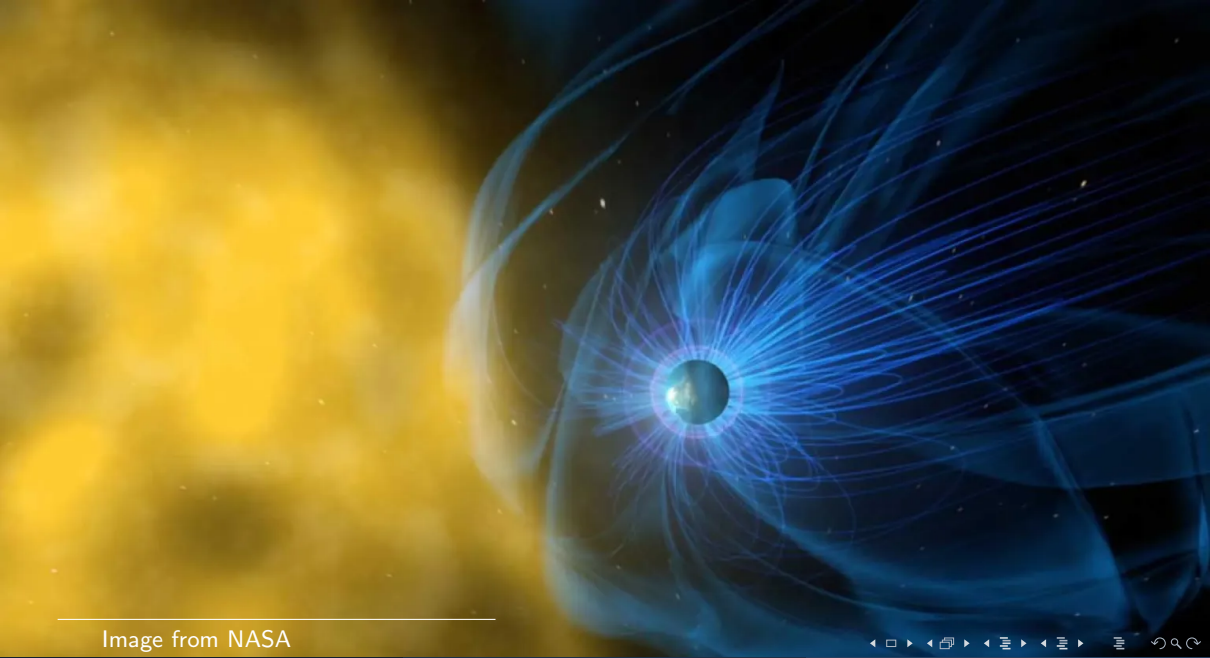
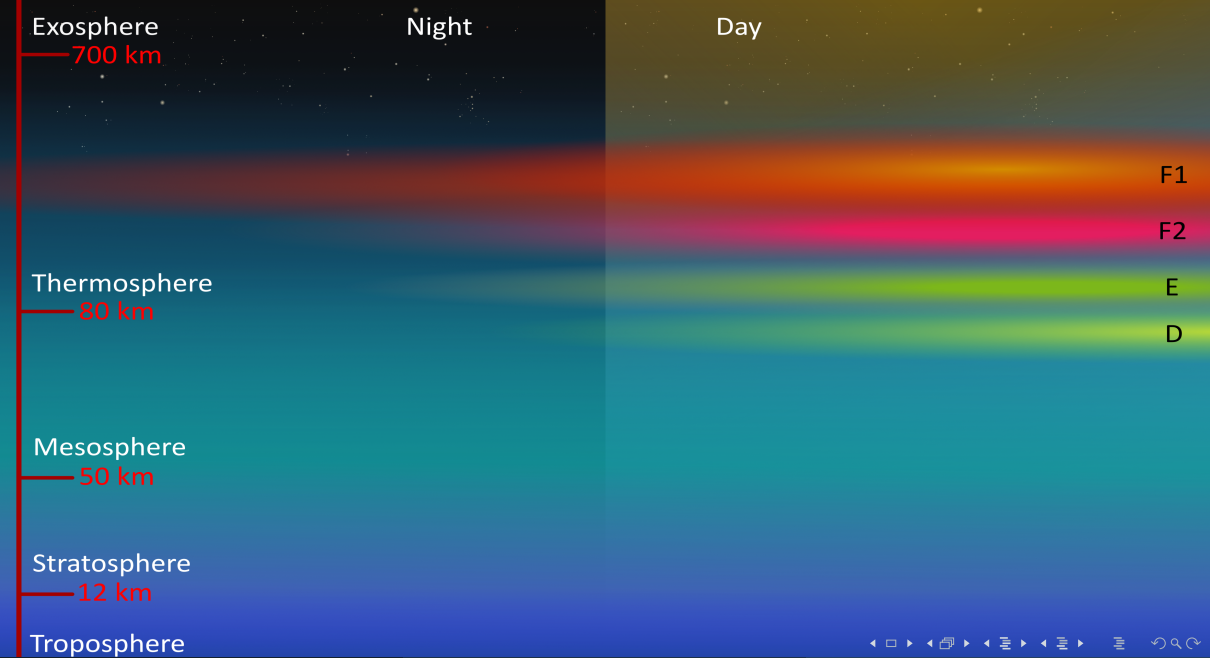


Image from NASA


Exosphere
700 km
Night
Day
F1
F2
E
D
Thermosphere
80 km
Mesosphere
50 km
Stratosphere
12 km
Troposphere

Sources of Ionospheric Variability

- Day/Night (Diel) Shift
- Solar Flares
- Auroral Substorms
- Geomagnetic Storms
- Atmospheric Gravity Waves (AGWs)
- Solar Eclipses

Terrestrial Atmospheric ITM Processes

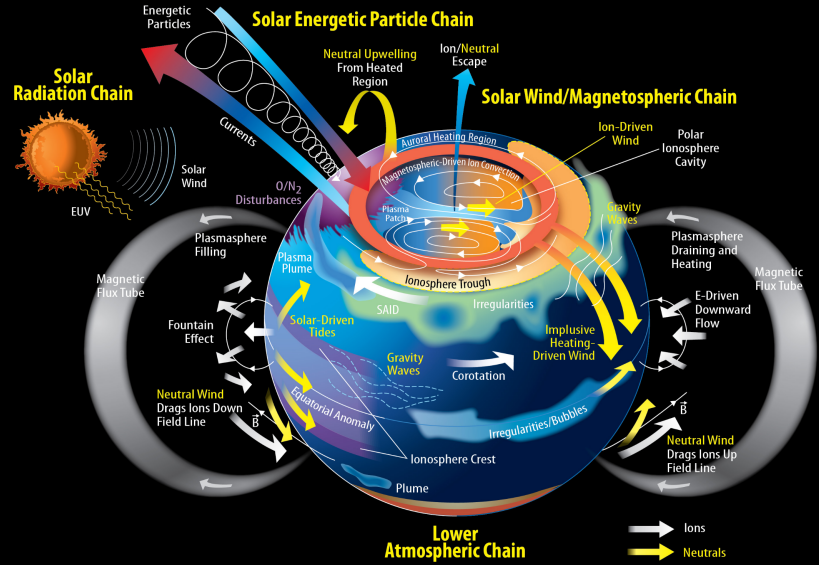


Image from NASA's Scientific Visualization Studio

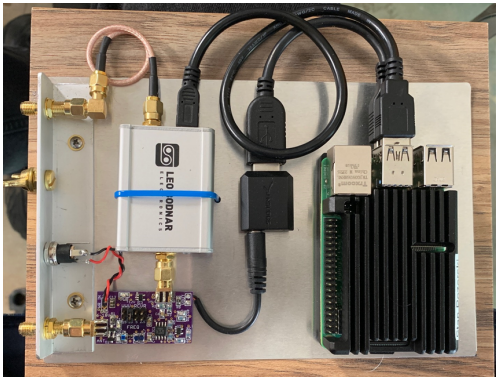
HamSCI

Personal Space Weather Station (PSWS) Projects Include:

1. Grape Receivers
2. Clementine SDR (WSPR Daemon)
3. Ground Magnetometers
4. FAIR Data

GRAPE

- GPS disciplined oscillator to allow accurate measurement of Doppler shift from WWV/WWVH
- Inexpensive and distributed
- Performs well in bunches
- Will ferment into something quite nice
- Includes original grapes, comercial hardware, and WSPR Grapes



The Grape 2

- Designed by John Gibbons, N8OBJ
- Three receiver channels
- Currently only around 30 units
- Records significantly more bandwidth than the Grape 1
- Integrates with an optional ground magnetometer station

Signal Doppler Shift

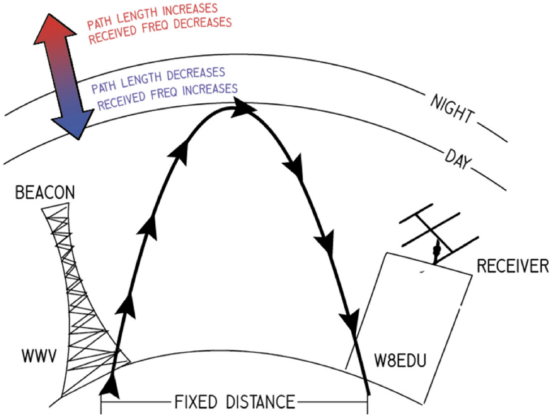


Image by K. Collins, KD8OXT

Signal Doppler Shift

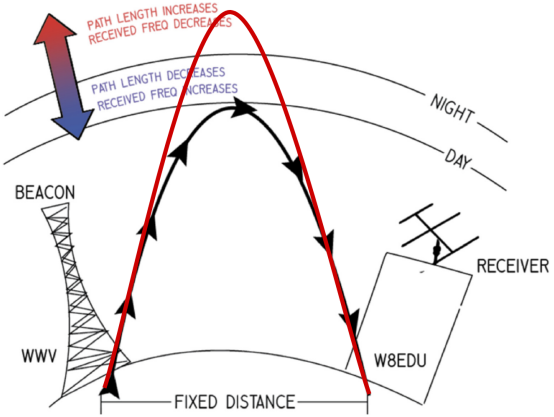


Image by K. Collins, KD8OXT

Signal Doppler Shift

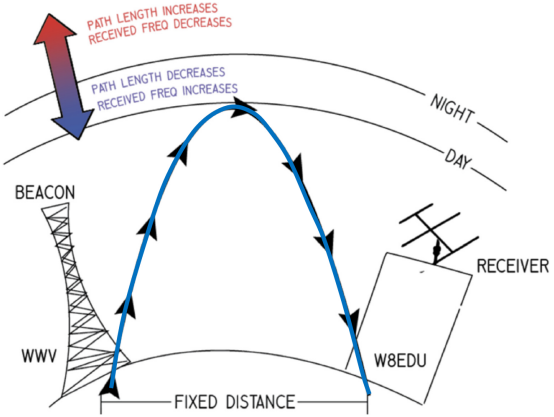


Image by K. Collins, KD8OXT

During a solar eclipse, there is a predictable time, path, and solar energy energy exposure. Solar radiation will still vary with season, maximum obscuration (annular, partial, or total), location, direction, and atmospheric state.

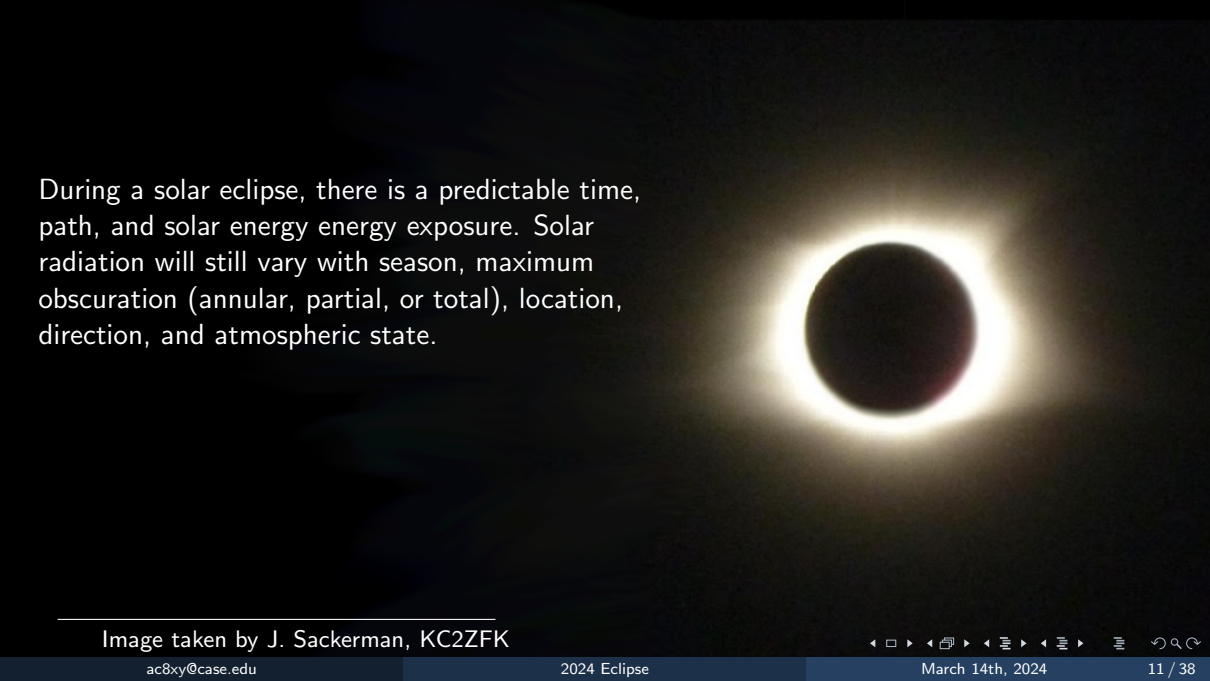
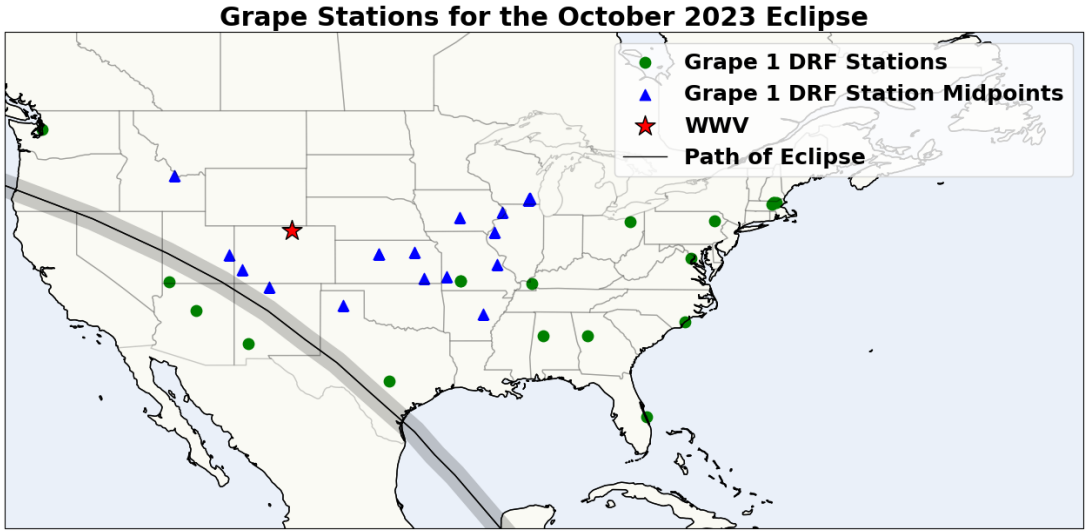
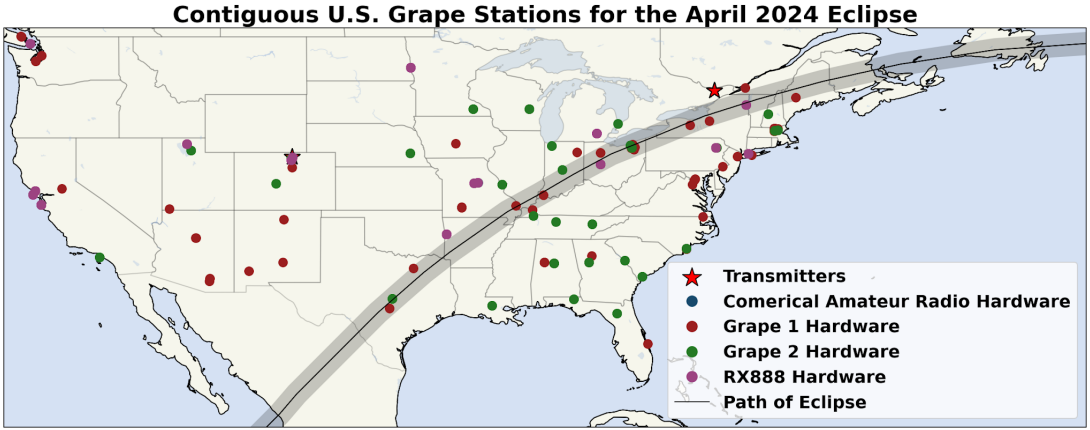


Image taken by J. Sackerman, KC2ZFK

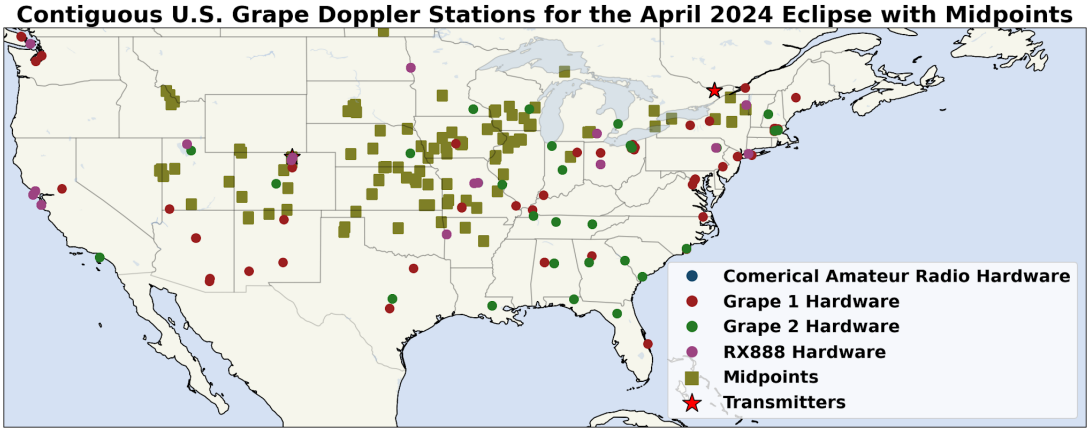
Grapes Deployed for the 2023 Eclipse



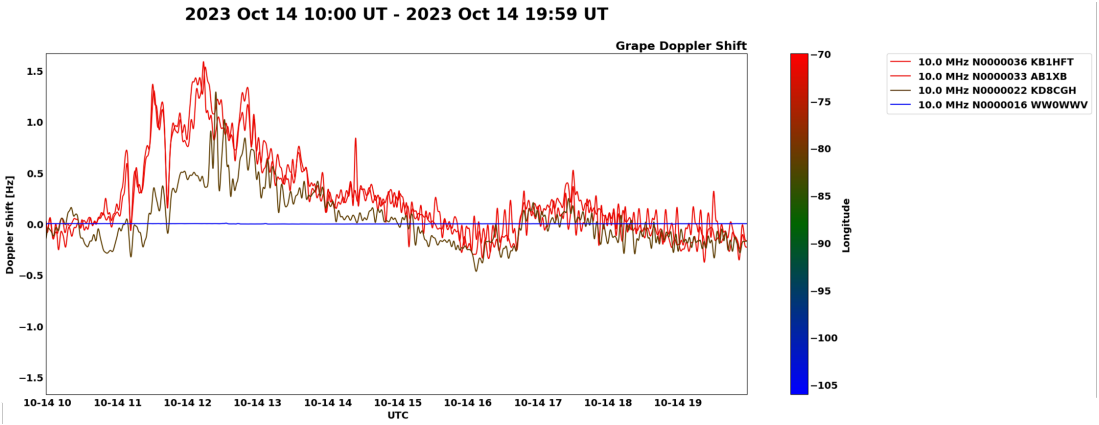
Grapes Deployed for the 2024 Eclipse



Grapes Deployed for the 2024 Eclipse



Grape 1 Classic Data



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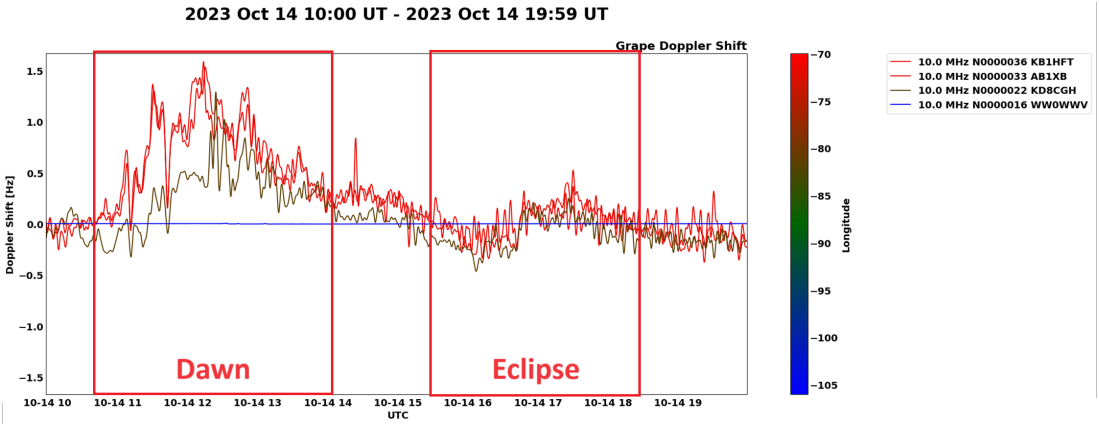
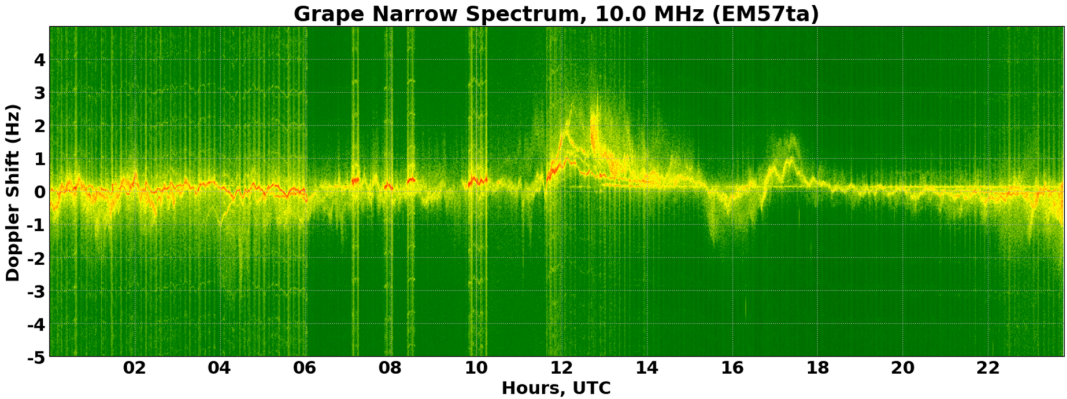


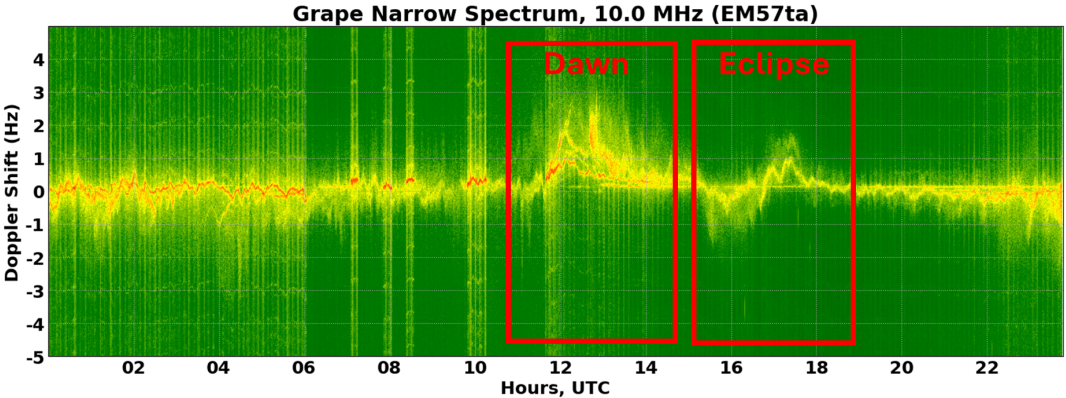
Digital Radio Frequency (Spectrum) Data

From N4TVC in Kentucky:



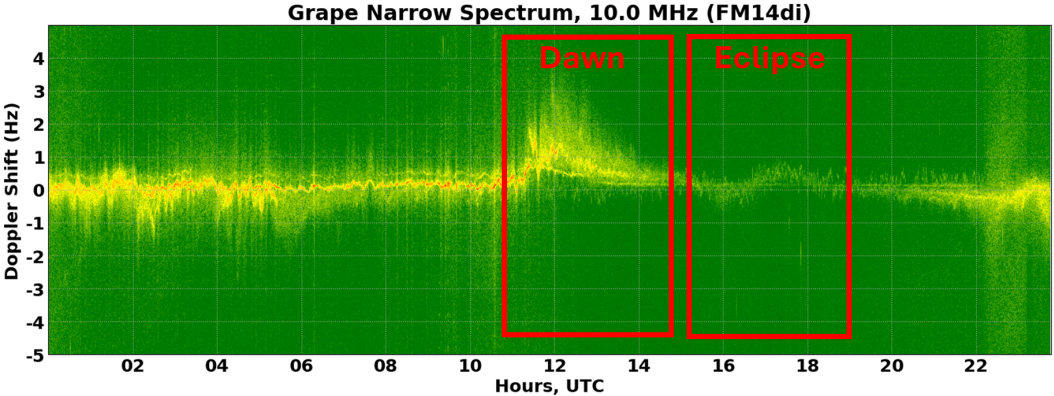
Dusk/Dawn vs. Eclipse Spectrum

From N4TVC in Kentucky:



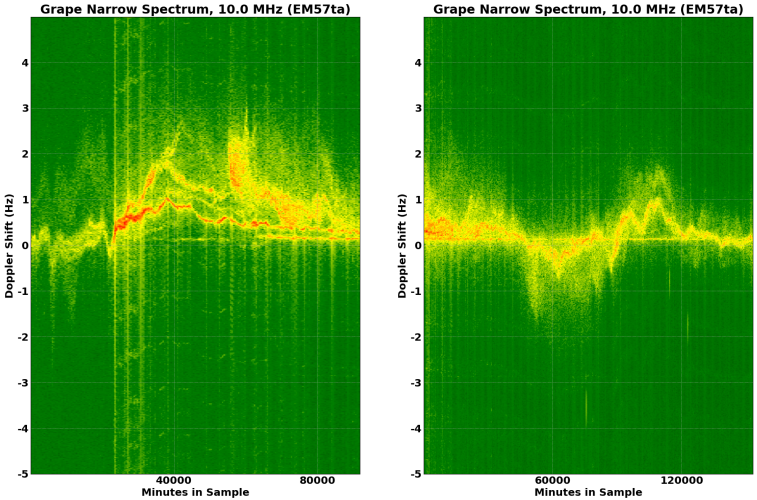
Dusk/Dawn vs. Eclipse Spectrum

From W1XP in North Carolina:



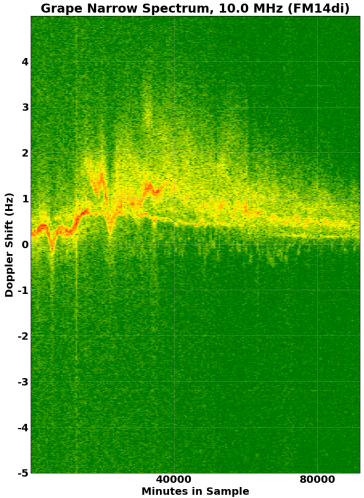
Dusk/Dawn vs. Eclipse Spectrum

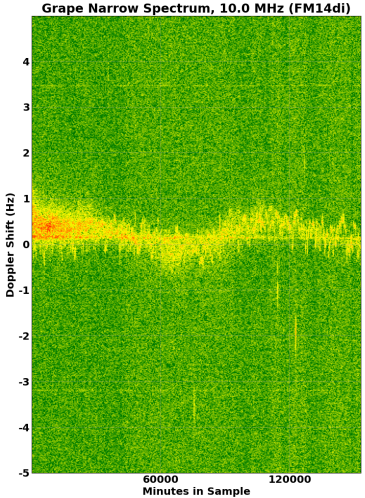
Let's take a closer look at N2TVC's data from Kentucky (Dawn and Eclipse):



Dusk/Dawn vs. Eclipse Spectrum

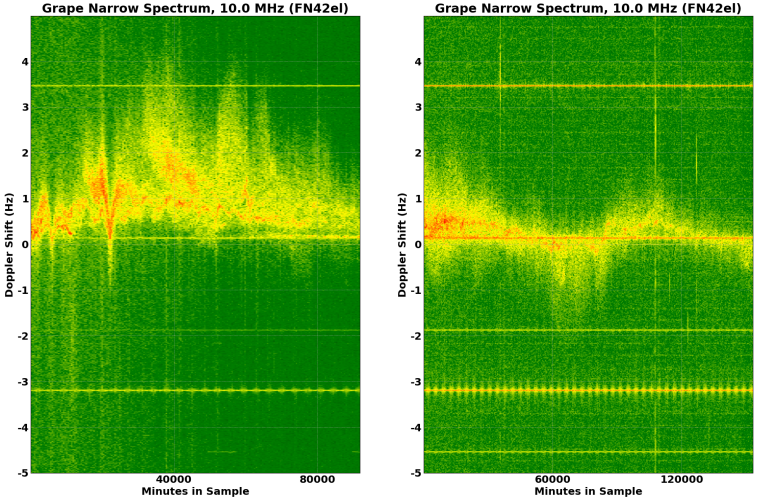
Let's take a closer look at W1XP's data from North Carolina (Dawn and Eclipse):





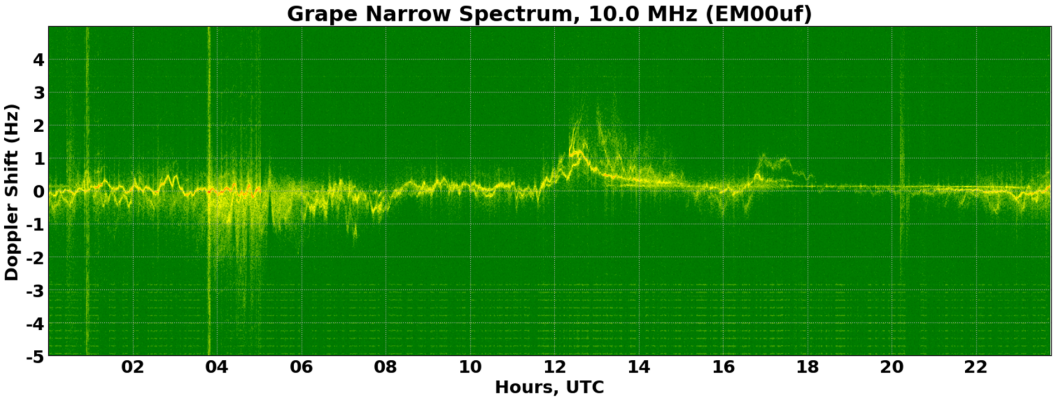
Dusk/Dawn vs. Eclipse Spectrum

Let's take a closer look at AB1XB's data from Massachusetts (Dawn and Eclipse):

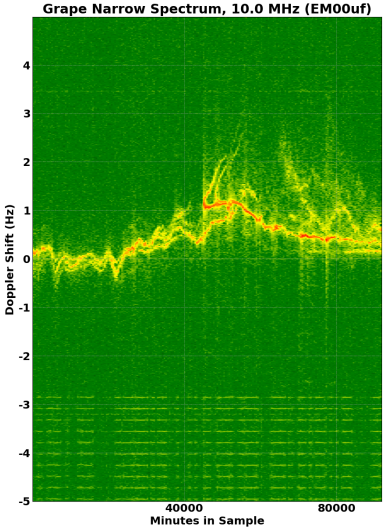


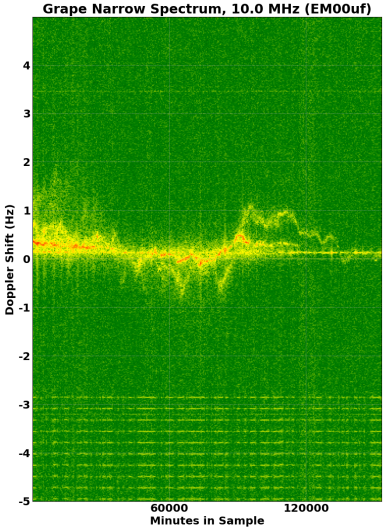
Multi-Path Signatures

data from K5TR near Austin, TX:



Multi-Path Signatures





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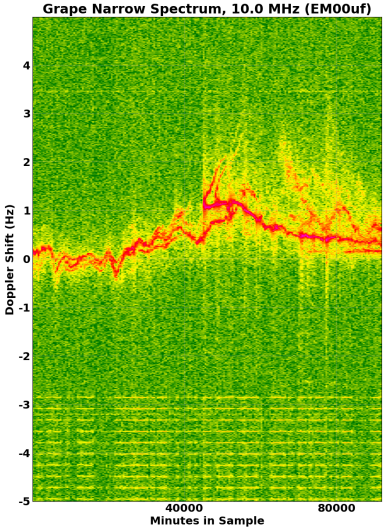
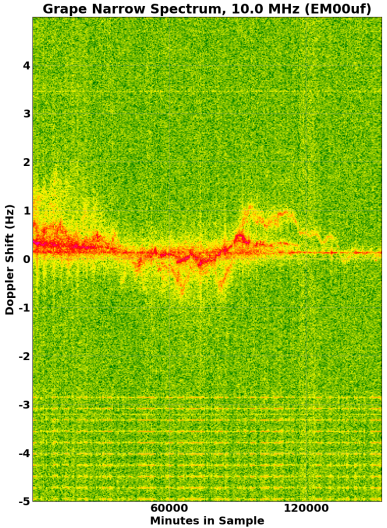
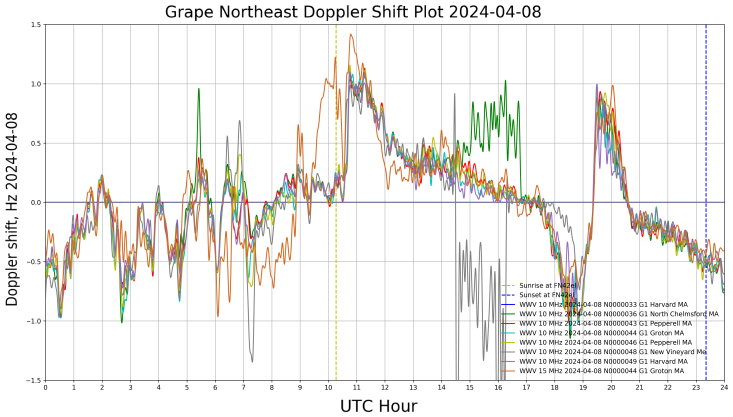
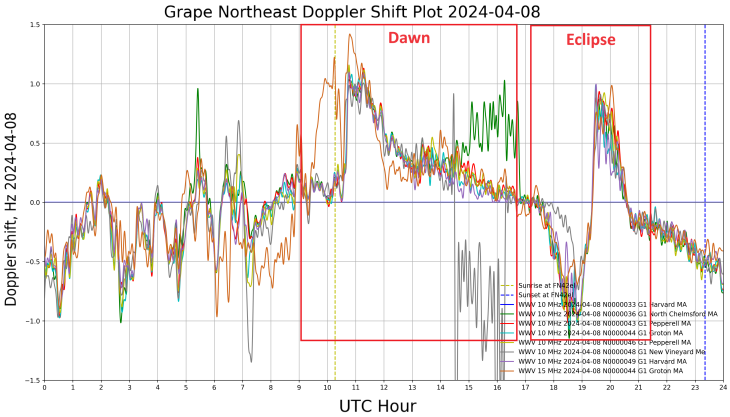




2024 Total Solar Eclipse Grape Data


Grape Northeast Doppler Shift Plot 2024-04-08
Doppler shift, Hz 2024-04-08
UTC Hour
Sunrise at FN42eh
Sunset at FN42eh
WWV 10 MHz 2024-04-08 N0000033 G1 Harvard MA
WWV 10 MHz 2024-04-08 N0000036 G1 North Chelmsford MA
WWV 10 MHz 2024-04-08 N0000043 G1 Pepperell MA
WWV 10 MHz 2024-04-08 N0000044 G1 Groton MA
WWV 10 MHz 2024-04-08 N0000046 G1 Pepperell MA
WWV 10 MHz 2024-04-08 N0000048 G1 New Vineyard Me
WWV 10 MHz 2024-04-08 N0000049 G1 Harvard MA
WWV 15 MHz 2024-04-08 N0000044 G1 Groton MA

2024 Total Solar Eclipse Grape Data


Grape Northeast Doppler Shift Plot 2024-04-08
Dawn
Eclipse
Doppler shift, Hz 2024-04-08
UTC Hour
Sunrise at FN42eh
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WWV 10 MHz 2024-04-08 N0000033 G1 Harvard MA
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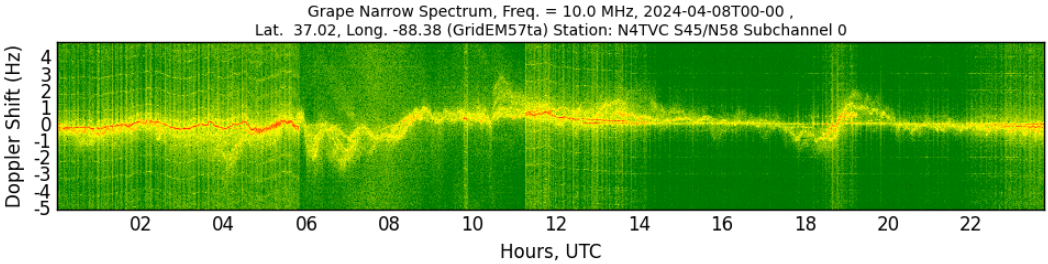
WWV 10 MHz 2024-04-08 N0000048 G1 New Vineyard Me
WWV 10 MHz 2024-04-08 N0000049 G1 Harvard MA
WWV 15 MHz 2024-04-08 N0000044 G1 Groton MA

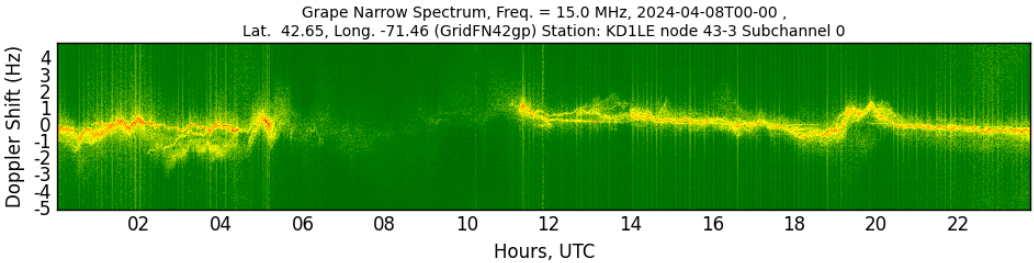
2024 Total Solar Eclipse Spectrum Data

From N4TVC in Kentucky:



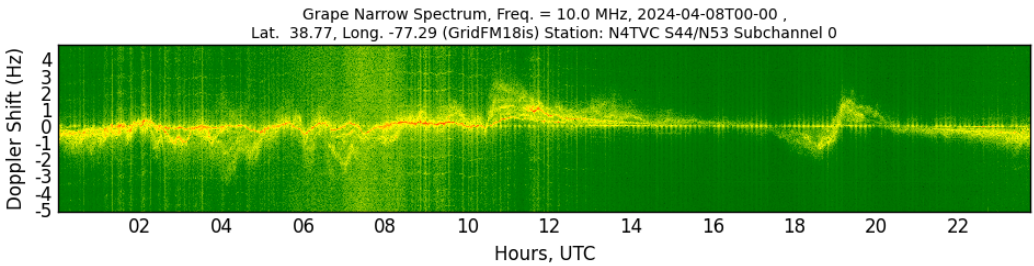
2024 Total Solar Eclipse Spectrum Data

From KD1LE in Massachusetts:



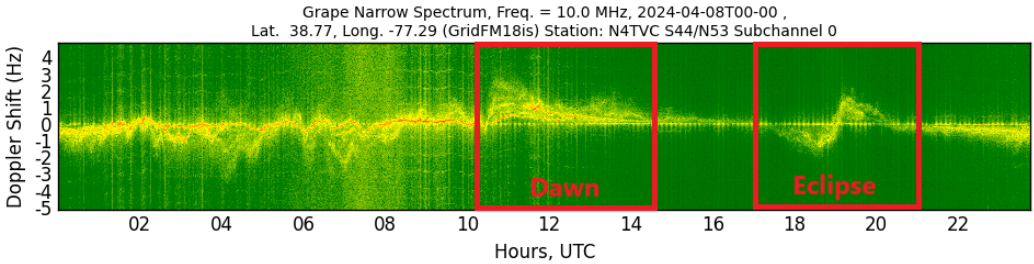
2024 Total Solar Eclipse Spectrum Data

From N4TVC in Virginia:



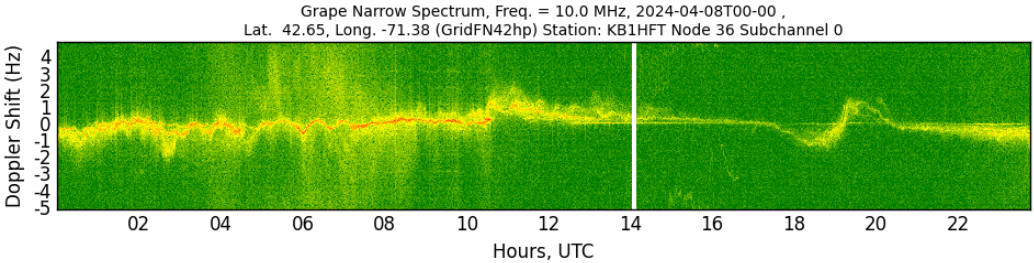
2024 Total Solar Eclipse Spectrum Data

From N4TVC in Virginia:



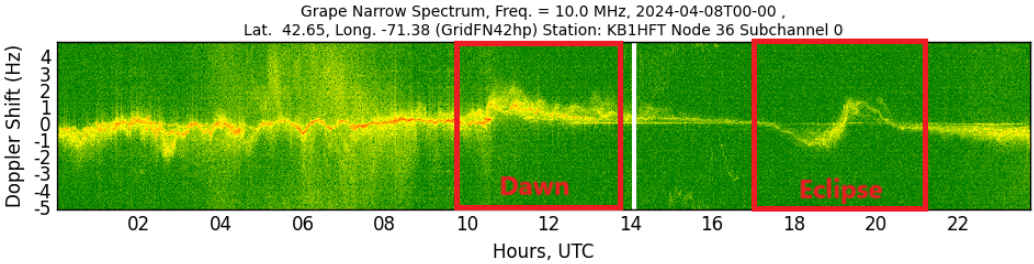
2024 Total Solar Eclipse Spectrum Data

From KB1HFT in Massachusetts:



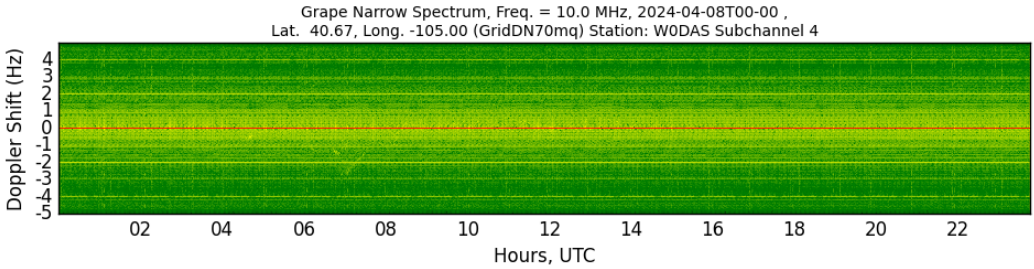
2024 Total Solar Eclipse Spectrum Data

From KB1HFT in Massachusetts:



2024 Total Solar Eclipse Spectrum Data

From W0DAS in Colorado:



Inverse Problems

- In a forward problem there is a known model, or transform, which can be applied to an input, x, and produces an output, b.
- In an inverse problem, we have the output value of a noisy problem and want to find the model (transform), represented by a matrix $A \in \mathbb{R}^{m \times n}$ such that for some data x we have an output b:

$$Ax + e = b$$

where e is an additive noise vector.

Hansen [2]

A Note on Notation

$A \in \mathbb{R}^{m \times n}$ is a matrix with m rows and n columns that maps a n-vector onto a m-vector.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Given a known transmission signal from WWV, x, and a received noisy signal, $b = Ax + e$, we want to reconstruct the ionospheric impact, A.

Terrestrial Atmospheric ITM Processes

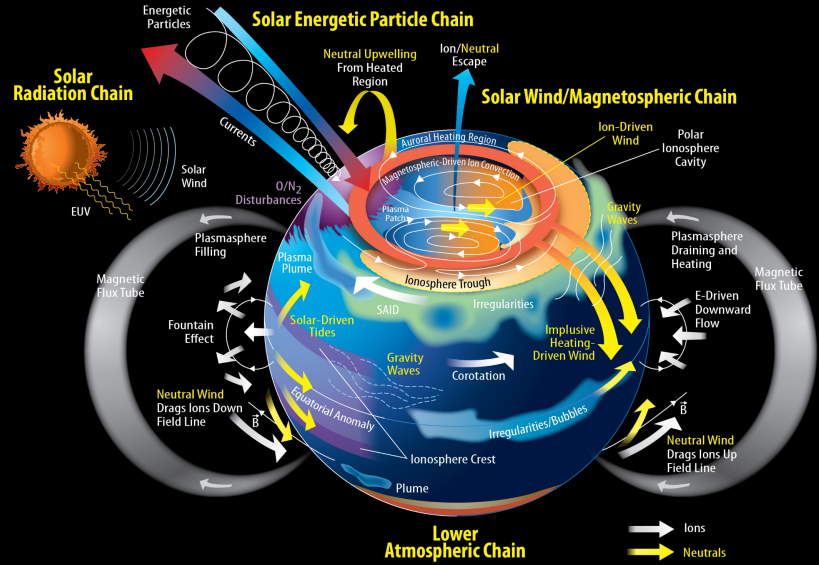


Image from NASA's Scientific Visualization Studio

Well-Posed Problem

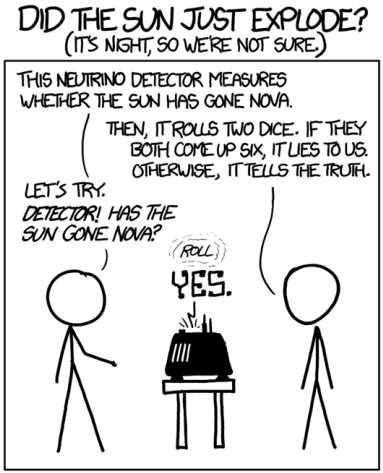
The usual criterion for a well-posed problem includes strict requirements for the solution of the problem, namely:

1. existence
2. uniqueness
3. stability

By their nature, inverse problems can be quite ill-posed, and usually require adjusting around such problems.

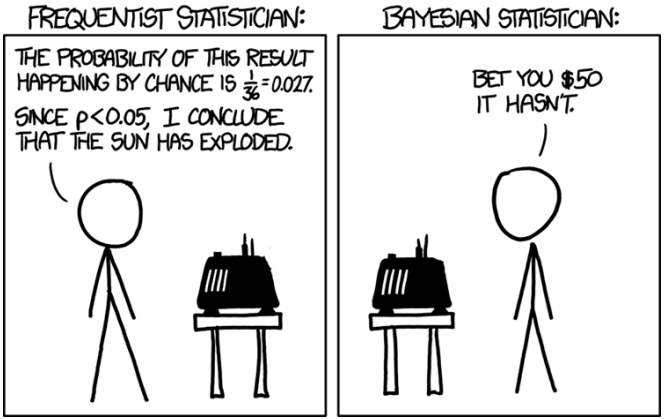
Hansen [2]

The Bayesian Mindset


DID THE SUN JUST EXPLODE?
(IT'S NIGHT, SO WE'RE NOT SURE.)
THIS NEUTRINO DETECTOR MEASURES WHETHER THE SUN HAS GONE NOVA.
LET'S TRY.
DETECTOR! HAS THE SUN GONE NOVA?
ROLL
YES.
THEN, IT ROLLS TWO DICE. IF THEY BOTH COME UP SIX, IT LIES TO US. OTHERWISE, IT TELLS THE TRUTH.

XKCD 1132

The Bayesian Mindset


FREQUENTIST STATISTICIAN:
THE PROBABILITY OF THIS RESULT HAPPENING BY CHANCE IS $\frac{1}{36} = 0.027$.
SINCE $p < 0.05$, I CONCLUDE THAT THE SUN HAS EXPLODED.
BAYESIAN STATISTICIAN:
BET YOU $50 IT HASN'T.

XKCD 1132

Bayes Formula

For events A and B which are sets of some outcomes, we say the probability of event A given that event B occurs (formally, $P(B) > 0$)

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

which arises intuitively from the notion that

$$P(A \cap B) = P(A \mid B)P(B)$$

In the formula

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

1. $P(A \mid B)$ is called the posterior distribution

Calvetti [1]

In the formula

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

1. $P(A \mid B)$ is called the posterior distribution
2. $P(B \mid A)$ is called the likelihood

Calvetti [1]

In the formula

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

1. $P(A \mid B)$ is called the posterior distribution
2. $P(B \mid A)$ is called the likelihood
3. $P(A)$ is called the prior distribution

Calvetti [1]

In the formula

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

1. $P(A \mid B)$ is called the posterior distribution
2. $P(B \mid A)$ is called the likelihood
3. $P(A)$ is called the prior distribution
4. $P(B)$ is called the marginal distribution

Calvetti [1]

In the formula

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

1. $P(A \mid B)$ is called the posterior distribution
2. $P(B \mid A)$ is called the likelihood
3. $P(A)$ is called the prior distribution
4. $P(B)$ is called the marginal distribution

Calvetti [1]

Priors

Using prior knowledge we can set up conditions for boundary conditions, noise distributions, and how smooth we expect the function to be.

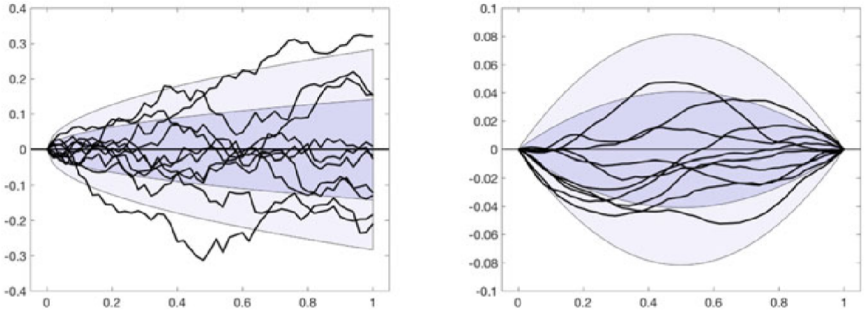


Image from Calvetti and Somersalo [1]

Bayesian Filtering

Imagine we have an indirect measurement for a physical phenomenon. For example,

1. If we try to find a hunter hidden in woodland and track their motions. We may not be able to locate the hunter, but we may be able to hear their hunting dog, which we assume stays within a certain radius of the hunter at all times. We can make a sequence of estimates based on our prior assumptions each time we make an indirect measurement.
2. The Grape receivers do not measure a direct height or ionization for the ionosphere, but paired with other assumptions we can use the Doppler measurement to make an uncertainty bound for the movement of an ionospheric layer at various points.

Calvetti[1]

Kalman Filter

The Kalman Filter is an iterative process that allows us to update our estimates.Consider a stochastic process $X_0, X_1, \ldots$ in $\mathbb{R}^n$ and an observation process $B_1, B_2, \ldots$ taking on values in $\mathbb{R}^m$ such that

$$X_t = FX_{t-1} + V_t$$
$$B_t = AX_t + W_t$$

where t is an iteration, $F \in \mathbb{R}^{n \times n}$ and $A \in \mathbb{R}^{m \times n}$, and

1. $X_0 \sim \mathcal{N}(\overline{x}_0, D_0)$
2. $V_t \sim \mathcal{N}(0, \Gamma_t)$
3. $W_t \sim \mathcal{N}(0, \Sigma_t)$
4. X_0, V_t, and W_t are mutually independent

Calvetti[1]

Kalman Filter

We call the prediction step values

$$\hat{x}_t = F\bar{x}_{t-q}$$
$$\hat{D}_t = FD_{t-1}F^T + \Gamma_t$$

where $\bar{x}_t = \hat{x}_t + K_t(b_t - A\hat{x}_t)$ and

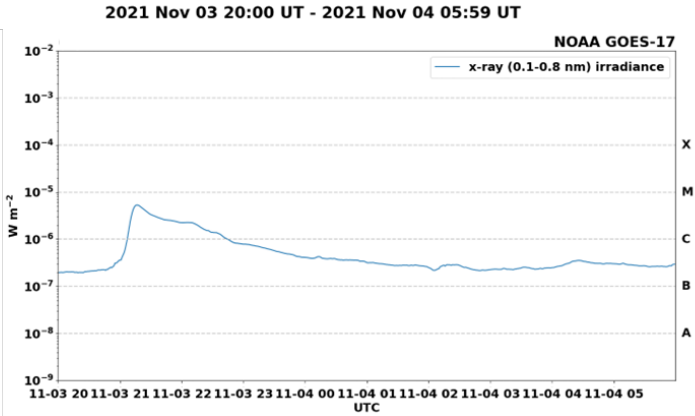
$$K_t = \hat{D}_t A^T (A\hat{D}_t A^T + \Sigma_t)^{-1}$$

is the Kalman Gain Matrix, and update according to

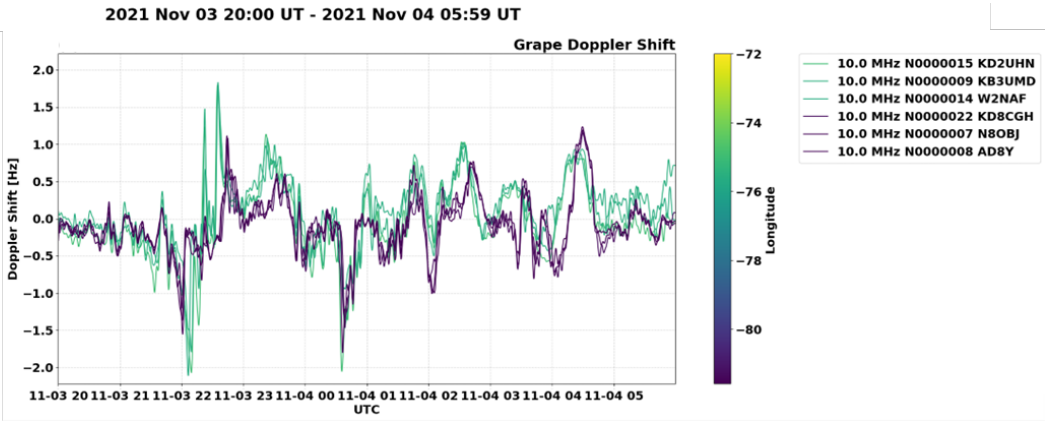
$$\overline{x}_t = \hat{x}_t + K_t(b - A\hat{x}_t)$$
$$D_t = \hat{D}_t - K_t A\hat{D}_t$$

Calvetti[1]

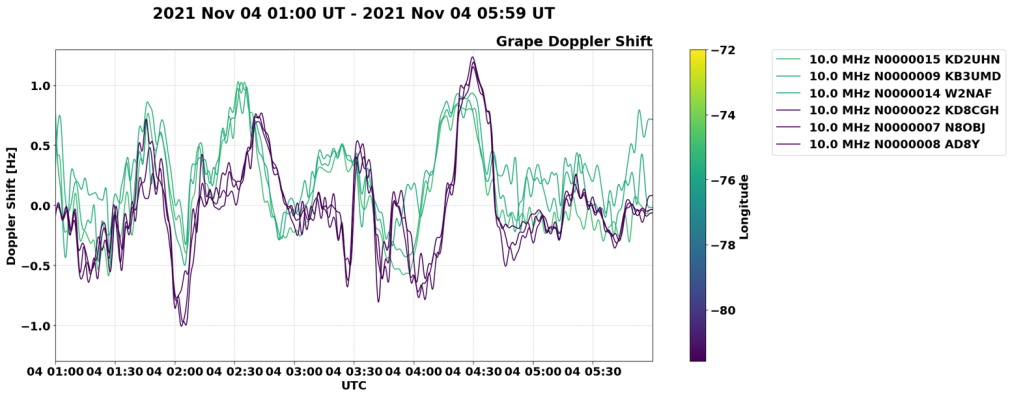
Traveling Ionospheric Distrubance

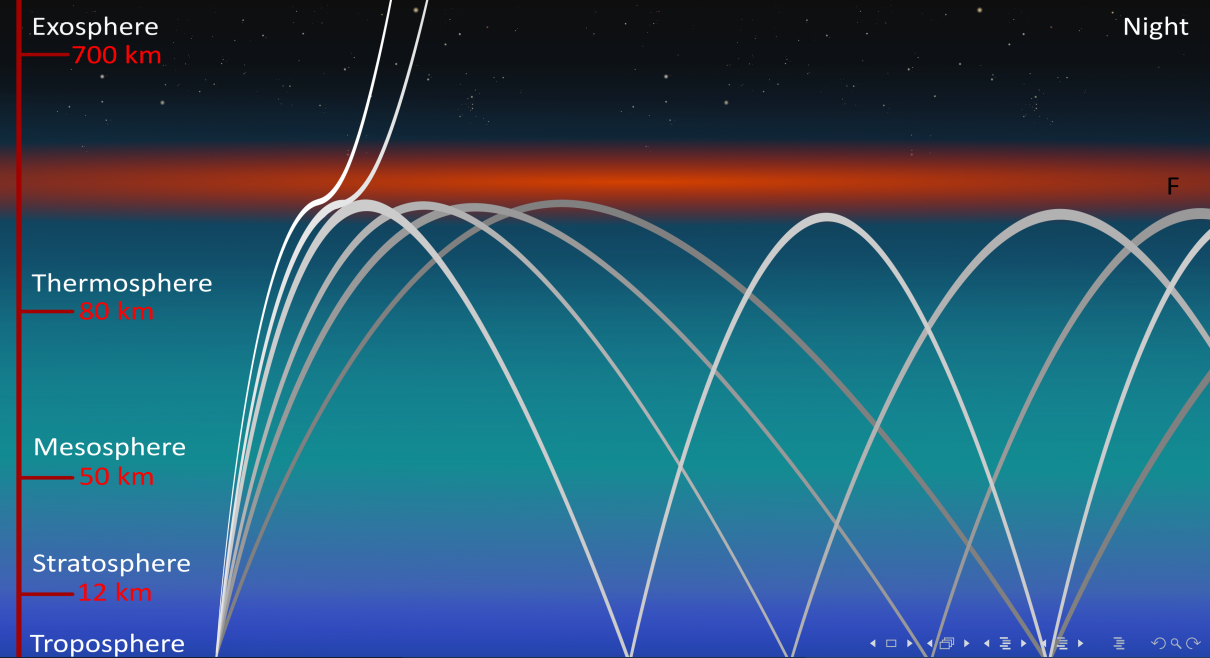


Traveling Ionospheric Distrubance

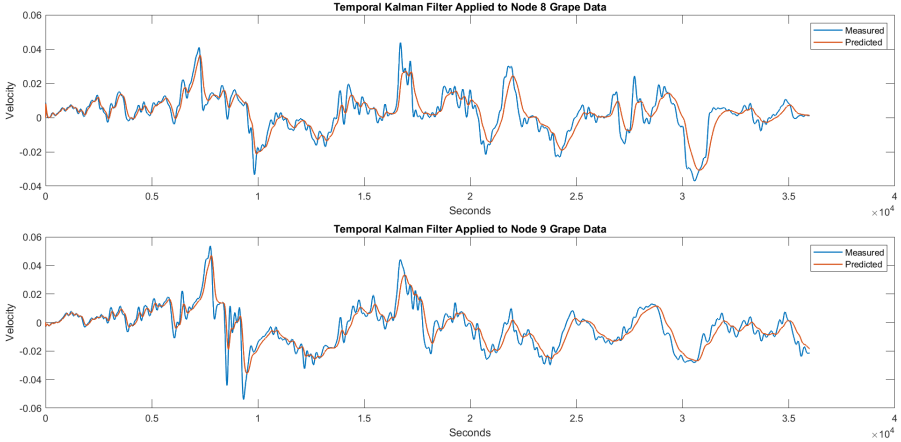


Traveling Ionospheric Distrubance




Exosphere
700 km
Night
F
Thermosphere
80 km
Mesosphere
50 km
Stratosphere
12 km
Troposphere

Example Kalman Process



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- HamSCI Lead
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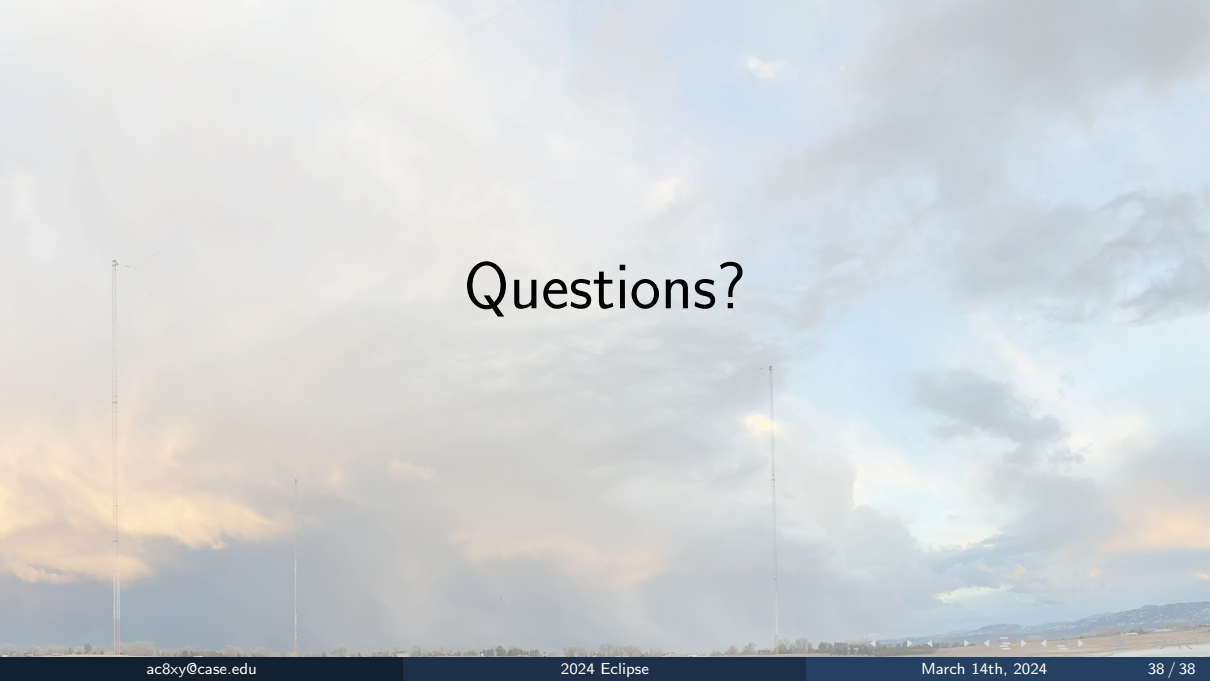
[2] P. C. Hansen, *Discrete Inverse Problems: Insight and Algorithms*, Fundamentals of Algorithms, Society for Industrial and Applied Mathematics, 2010.


Questions?